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353. Proposed by CLIFFORD N. MILLS, Brookings, North Dakota.

A uniform beam of oak, 10 feet long, 15 inches deep and 10 inches wide, sustains, in addition to its own weight, a load of 5,000 lbs. placed at the center. Find the greatest bending moment and the greatest stress in the fibers. Take the specific gravity of oak as .934.

NUMBER THEORY.

270. Proposed by GERSHOM N. CARMICHAEL, Urbana, Ill.

Does there exist a fraction p/q in its lowest terms such that the ratio of the sum of the divisors of p to the sum of the divisors of q is equal to p/q? Give a method of finding such fractions not in their lowest terms.

271. Proposed by HORACE OLSON, Chicago, Ill.

Prove that if x, y, z, u, v, and w are integers such that $x^2 + y^2 = u^2$, $x^2 + z^2 = v^2$, $y^2 + z^2 = w^2$, then the product $xyzuvw$ is divisible by 518400.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

474. Proposed by A. A. BENNETT, University of Texas.

Show that the value of the infinite continued fraction, all of whose coefficients are unity,

$$1 + \frac{1}{1+}\,\frac{1}{1+}\cdots, \quad \text{is} \quad \tfrac{1}{2}(1+\sqrt{5}).$$

Also find an explicit algebraic formula for the nth convergent.

Solution by C. C. Yen, Tangshan, North China.

1. Let x denote the value of the continued fraction. Then,

$$x = 1 + 1/x, \quad \text{i. e.,} \quad x^2 - x - 1 = 0,$$

therefore,

$$x = \tfrac{1}{2}(1 + \sqrt{5}),$$

where the sign for the radical is positive, since x is evidently positive.

2. Let p_n/q_n denote the nth convergent. Then

$$p_1 = 1, \qquad q_1 = 1, \qquad p_2 = 2, \qquad q_2 = 1. \tag{I}$$

Also, since all the coefficients are unity,

$$p_n = p_{n-1} + p_{n-2}, \qquad q_n = q_{n-1} + q_{n-2} \qquad (n = 3, 4, 5, \cdots). \tag{II}$$

And, it follows, therefore, from I and II,

$$q_n = p_{n-1} \qquad (n = 2, 3, 4, \cdots). \tag{III}$$

Now consider the series

$$1 + x + 2x^2 + p_3x^3 + p_4x^4 + \cdots + p_nx^n + \cdots,$$

where the coefficients p_n satisfy II. It is a recurring series whose scale of relation is $1 - x - x^2$, and whose generating function is $1/(1 - x - x^2)$. Hence, p_n is the coefficient of x^n of the expansion of this function; and, by III, q_n is the coefficient of x^{n-1} of the same expansion.

If we put $\alpha = -\frac{1}{2}(1 + \sqrt{5})$, $\beta = -\frac{1}{2}(1 - \sqrt{5})$, we get

$$\frac{1}{1 - x - x^2} = \frac{1}{\alpha - \beta}\left(\frac{1}{\alpha - x} - \frac{1}{\beta - x}\right),$$

whence the general term of the expansion of $1/(1 - x - x^2)$ is

$$\frac{1}{\alpha - \beta}\left\{\left(\frac{1}{\alpha}\right)^{n+1} - \left(\frac{1}{\beta}\right)^{n+1}\right\}x^n = \frac{1}{\alpha - \beta}\left\{\frac{\beta^{n+1} - \alpha^{n+1}}{(\alpha\beta)^{n+1}}\right\}x^n.$$